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REVIEW

One hundred years of Lamb's Hydrodynamics

The year 1879 marked the publication by Cambridge University Press of Horace Lamb's 258 page *Treatise on the Motion of Fluids*, the author's preface being date-lined May 16th, Adelaide. Subsequent editions which the author tells us were 'largely remodelled and extended' appeared, roughly one per decade until the 738 page sixth in 1932, under the title *Hydrodynamics* so familiar to us when associated with Lamb's name.

The copy of the first edition available to me shows the date of publication as 1879 but in fact includes a C.U.P. Catalogue dated 1882 which gives the price as twelve shillings. One may compare this price with Thomson & Tait's contemporary *A Treatise on Natural Philosophy*, volume 1 for sixteen shillings and also with the current price of Lamb's sixth edition, remarkably still available at £13.50 (in hard covers). This represents little more than 3% inflation over the 100 years, though to one who had his Lamb in the early thirties (bound in half calf as a College prize and regrettably a fifth edition only weeks before the much revised sixth appeared) still rather frightening. (The sixth edition was priced at 45 shillings on publication in 1932; this amounts to about $2\frac{1}{2}$% inflation from 1879 to 1932 and about 4% from 1932 to 1978.)

I used the word 'remarkably' in the previous paragraph in its literal sense but one can't say that the survival is surprising when one considers how much information basic to modern applications of classical fluid mechanics it still contains. Much of what G. I. Taylor wrote in his obituary of Lamb (1849–1935) in *Nature* (1935, 255–7) remains true today. After giving the background by remarking that in Lamb's young days 'The proper way in which a lecturer could make known any theorem which he might discover in his teaching was to set it as a tripos question' and adding 'The science of hydrodynamics was at that time concerned almost entirely with an idealised, non-existent fluid which moved only in irrotational motion, without vorticity, and was thus well adapted for tripos questions', Taylor went on:

> Lamb, in his first course of lectures on hydrodynamics, given at Trinity in 1874, broke new ground when he gave an account of Helmholtz's great work on vortex motion. The substance of these lectures was published in 1878 (*sic*) as a *Treatise on the Motion of Fluids*. This book, of some 250 pages, expanded in subsequent editions until as *Hydrodynamics* it covered some 700 pages. During its long career, which is still in full vigour, it has become the foundation on which nearly all subsequent workers in hydrodynamics have built. The long-continued supremacy of this book in a field where much development has been taking place is very remarkable, and is evidence of the complete mastery which its author retained over this subject throughout his life.
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> It is of interest to notice through the various editions of *Hydrodynamics* the continually increasing stress which is laid on the physical side of hydrodynamics. In the first edition (1879), the mathematical consequences of the conception of an ideal fluid are systematised and generalised in a form which is aesthetically very satisfying, and special problems are treated mostly as exercises of the type which

occur in the tripos. In subsequent editions, problems are treated more from the point of view of their intrinsic interest as illustrating natural phenomena or experimental conditions. Numerical values are given for results which at first appeared only in symbolical form. Motions such as turbulent flow, which even now defy exact mathematical treatment, are discussed, in the later editions, in the partial and incomplete forms which they had attained at the time of publication. New developments have been brought into the scheme of the book, and it is this continuous growth as an organic whole that has enabled Lamb's *Hydrodynamics* to be still, after fifty-five years of life, the best book on the subject.

This assimilation of new developments into successive editions is also commented on by the (unnamed) writer of Lamb's obituary in *The Times*. He says, with however an unmistakable note of caution in his parenthetic comments,

> The sixth revision, in 1933, although the work of a man of 83 and not wholly brought into line with the latest ideas, shows no signs of being superseded.

The *Times* writer also analysed Lamb's general outlook in the following terms:

> The primary aim of science in Lamb's view was to explore the facts of Nature, to ascertain their mutual relations, and to arrange them so far as possible in a consistent and intelligible whole. The material effects came later, if at all, and often by a very indirect path. The mathematician's task, to his mind, had an aesthetic character. He took delight in the comparison of a well-ordered piece of algebraic analysis with a musical composition, and bemoaned the passing of the scientific memoir... For himself he could not draw a sharp distinction between pure and applied mathematics, and was never tired of quoting Fourier's saying: 'L'étude approfondie de la nature est la source la plus féconde des découvertes mathématiques'. It was perhaps this realization which made him look somewhat askance at the 'arithmetization of all mathematics', when that was the vogue.

I have to confess that the source of this last quotation inside this quotation is not familiar to me and that I find this embodiment of Lamb's sentiment obscure. Unless I misunderstand it completely it suggests that Lamb, master that he was of analytical methods, would not have welcomed, had they been available earlier, the advances that numerical analysis and computing have now made possible. That I find difficult to believe, particularly in view of the evolutionary trends towards numerical results, remarked on by Taylor, in successive editions.

Let us look at the actual changes which took place between the first and sixth editions. The first had nine chapters and four notes; the sixth twelve chapters, nine of which had the same (or almost the same) headings as the original. The three additions in the sixth edition are: Chapter V, on 'Irrotational Motion of a Liquid: Problems in Three Dimensions' (though this includes part of the original Chapter V on 'Motion of Solids through Liquids'); Chapters VIII on 'Tidal Waves' and IX on 'Surface Waves', which replace a single chapter (VII) on 'Waves in Liquids', and Chapter XII on 'Rotating Masses of Liquid', which has no counterpart in the first edition. It must not be deduced from this comparison that the content of any of the chapters remains unaltered though it warrants comment that a sizeable part of the original was subsumed with many of the alterations being in the shape of additions. One should also note in this context that the 10% of the original devoted to waves in liquids has become 30% of the much larger sixth edition. The way the work developed through the various editions is summarized in a contemporary review of the sixth edition by Goldstein (*The Mathematical Gazette*, 1933, pp. 215–16), who starts by referring to the first edition as 'a smallish volume' and then says:

> The second edition, published sixteen years afterwards, was more than twice the size of the first, and bore the altered title *Hydrodynamics*. By the third edition, published in 1906, both the outward appearance and the general character of the book were determined. The classical theory of the motion of an ideal, inviscid, incompressible fluid, and the theory of waves, were being treated by a master.

Goldstein then goes on to list as additions in the sixth edition:

> The equations of motion in general orthogonal coordinates, electrical analogies, theorems and examples on the calculation of the forces and moments on an immersed cylinder in two-dimensional flow, a slight extension of the discussion of the Joukowski aerofoil, recent important theoretical work on tidal motion, waves of finite amplitude of permanent type, the influence of viscosity on sound waves of permanent type, the generation and maintenance of waves by wind, changes and additions to the discussion of atmospheric waves, a reference to the drag formula for a cylindrical obstacle with a Kármán street in the wake, a reference to, and criticism of, the asymptotic theory of Oseen, a short discussion of dynamical similarity, the boundary layer equations for steady motion in two dimensions, and some discussion of the flow along a flat plate. One of the most important additions is, perhaps, the discussion of the influence of compressibility on the flow past an obstacle.

To this one may add that superficially an obvious change in presentation apart from typography and layout lies in the altered use of the operators d, ∂ and D in conformity with the notation of their respective times. Thus ∂ of the original becomes D and d is replaced by ∂ in the notation now familiar to us all. (It is perhaps worth a parenthetic comment that vector notation was becoming established only about the time of the sixth edition and finds no significant place in it.) An interesting measure of change can also be obtained from the lists of authors cited; a sizeable proportion of those listed in the sixth were not even born when the first was published – H. Jeffreys and G. I. Taylor to mention but two. It is an illuminating exercise for the reader to take this thought one stage further and see through these references how the subject stood initially and subsequently developed. Space prevents the elaboration of this point; suffice it to remark that the work of Taylor referred to included, in addition to his famous rotating cylinders, a number of the calculations (referred to by Goldstein, above) of forces and moments in irrotational flow.

In this connexion a comment by Lamb in the Preface to the sixth edition is noteworthy. After remarking on the pains that had been taken (as in previous editions) to make due reference in footnotes to authorities, he provides an interesting sidelight into the perfection he sought when he comments 'but it appears necessary to add that the original proofs have often been considerably modified in the text.' Most people would agree, now as then, that substantial benefits in presentation do in fact accrue from these modifications – the diffraction problem for a semi-infinite plane, first treated by Sommerfeld, is a case in point.

We may start a critical appraisal of Lamb's work by examining what points the writer of the *Times* Obituary (quoted above) probably had in mind when he spoke of it being 'not wholly brought into line with the latest ideas'. Reference to the review by Goldstein and to one in *Nature* by Jeffreys (1933, pp. 313–314) may provide us with clues. Jeffreys, after drawing a parallel between the evolution of a species and the development of a book through successive editions and commenting both on the differences between real and classical fluids and the emphasis placed by Lamb on the classical fluid, says

> Further, though classical hydrodynamics is scarcely ever in exact agreement with the facts, it is often an excellent approximation; for example, the whole of the chapters on tidal waves, surface waves, and waves of expansion are still valid in a viscous fluid. Still, seeing that the fundamental postulates of classical hydrodynamics are wrong, there is a definite problem in explaining why its results are ever right. The attention given to this question is somewhat casual... The proposition that vorticity in a real fluid cannot originate in the interior, but must be diffused inwards from the boundary, is given in small type on p. 578; I should prefer to state the result in the form of a modification of the circulation theorem, but even in the form given to it by Lamb it could have been made the basis of an explanation of why so much of the earlier part of the book has physical value. Again, much attention is given to vortex motion in Chapter VII; but we might have expected to find somewhere an explanation of how isolated vortices come to exist. Many results are given that have a bearing on these questions, but they are not co-ordinated in such a way as to bring out their fundamental importance.

Jeffreys is also very critical of the continued inclusion of the account of Reynolds' theory of the stability of laminar flow since, as he says, 'it proves nothing at all'.

Goldstein not surprisingly makes many of the same points as Jeffreys, though occasionally with somewhat different emphases. For instance he says, 'Moreover, Sir Horace Lamb can rightly claim in his preface to the new edition that classical hydrodynamics has been found to have a widening field of practical applications. Nevertheless, these applications mostly require the inclusion of circulation and vorticity, for the explanation of whose existence recourse must be made to viscosity and boundary layers; and Sir Horace Lamb has nowhere included an adequate discussion of the production of circulation and vorticity in fluids of small viscosity.' In this connexion both Goldstein and Jeffreys are very critical of Lamb's account in §79 of the flow of real fluids past projecting corners. Goldstein gives in some detail the now universally accepted argument that it is viscosity, no matter how small, rather than the cavitation explanation adopted by Lamb, which is the main cause of the difference between the observed phenomena and those calculated in ideal fluid theory. (There is no doubt that Lamb recognized that viscous effects had some importance but that he failed to seize on their overriding nature in this problem.) Goldstein, after quoting Lamb's remarks on the influence of viscosity as he (Lamb) saw it, then goes on: 'This, combined with some new remarks on p. 687 concerning separation of the boundary layer from curved surfaces, could easily be expanded into a complete explanation according to the boundary layer theory of the observed phenomena; and such an explanation could, indeed, have served so to join classical and modern hydrodynamics as to make them one continuous whole.'

Finally Goldstein sums up his view of the chapters on real fluids by saying:

> Although within a single chapter, even with the valuable additions now made, it is no longer possible to give a completely adequate account of our present state of knowledge of viscous and turbulent flow, and there are several matters which one would certainly have liked to have seen more fully treated, yet it is a matter for admiration how much is included, and how clearly a difficult subject is presented.

Looked at today, forty years on, the criticisms quoted lose no force. If one were to add one now, bearing fully in mind the material then available, it would I think have to concern the impact of thermodynamics on hydrodynamics. In this respect Lamb does not appear to have had the same mastery as elsewhere though it may be an exaggeration to say, as I have heard it said, that Lamb didn't know any thermodynamics. What is said in §284 of the work of Rankine (1870), Hugoniot (1887–9), Hadamard (1903) and Taylor (1910) about a postulated discontinuity (before the description shock wave had been adopted) in compressible flow seems at first sight to be a definite rejection of the Rankine–Hugoniot energy condition. As the discussion proceeds in §284 (and later in §360*a*) the means to resolve the difficulty begin to emerge but it doesn't make for an easy exposition or lead to a satisfying or clear conclusion. Certainly one couldn't recommend it as introductory reading on shock waves; the almost contemporary article by Taylor & Maccoll in volume 3 of Durand's *Aerodynamic Theory* (1935) drawing on the same material shows what could be done. I suppose Lamb's difficulty can be traced back to his introduction in §10 of intrinsic (internal) energy without specific reference to its property as a function of state; in shock waves the difficulty is compounded by an apparent fear of using the concept of entropy. On the other hand let it be said that in the discussion of viscosity in gases (§358) the difference between the pressure defined as minus the mean of the principal stresses and the thermodynamic pressure is correctly related to the second coefficient of viscosity, though the hypothesis is then made that the latter is zero.

Even when all the criticisms have been made the fact remains that, as remarked earlier, it is because Lamb contains so much information basic to modern applications of classical hydrodynamics that it has had this remarkable staying power. Not only are the fundamentals there, but accounts of judiciously chosen applications, for instance tides and surface waves, warrant careful attention from today's student. Seen historically it represents a tour de force for, in addition to its use as a text book, it pointed the way forward for hydrodynamics in the thirties and has been one of the most potent, perhaps *the* most potent, influence in the burgeoning of the subject which has taken place since.

That there has been such a burgeoning means in turn that no work within a single pair of covers could in the seventies do what Lamb's did in the thirties. Limiting oneself to the essentials one needs to include appropriate parts of *Modern Developments in Fluid Dynamics* (Editor Goldstein), Batchelor's *An Introduction to Fluid Dynamics*, Lighthill's *Waves in Fluids* and Whitham's *Linear and Non-Linear Waves* in a reading list which in the thirties could have been limited to Lamb's *Hydrodynamics*. Should Lamb's book itself appear in today's list? I think it should as an elegant introduction and, more than that, its clarity of presentation makes it a rewarding treasury.

One topic which I have not included in my list is turbulence. Here, too, much has been achieved in the meantime not least by Taylor and the Cambridge school. I hope however that in this context of turbulence I won't be thought inappreciative if I leave the last word to Lamb. He is reputed to have said that when he finally came face to face with his Maker he would seek clarification of the mysteries of quantum theory and turbulence, adding that he had reasonable hopes of being satisfied about quantum theory. Might he have said the same today?

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